

University of Wisconsin - Madison

MAD/PH/803
RAL-93-085
October 1993

RGE RESULTS FOR SUPERSYMMETRIC GUTS*

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ABSTRACT

The scaling behavior of gauge couplings and fermion Yukawa couplings in the minimal supersymmetric model is discussed. The relevance of the top quark Yukawa coupling fixed point in establishing the top quark mass is described. The evolution of mixing angles is presented.

1. Gauge Coupling Evolution

The one- and two-loop renormalization group equations (RGEs) can be written for general Yukawa matrices as

$$\frac{dg_i}{dt} = \frac{g_i}{16\pi^2} \left[b_i g_i^2 + \frac{1}{16\pi^2} \left(\sum_{j=1}^3 b_{ij} g_i^2 g_j^2 - \sum_{j=U,D,E} a_{ij} g_i^2 \text{Tr}[\mathbf{Y}_j \mathbf{Y}_j^\dagger] \right) \right], \quad (1)$$

with $\mathbf{Y}_j \equiv \mathbf{U}, \mathbf{D}, \mathbf{E}$ (the Yukawa coupling matrices) and $t = \ln \mu/M_G$. The low-energy values of the gauge couplings g_1 and g_2 lead to a prediction for $g_3[1]$ via the hypothesis of a grand unified theory (GUT). For the supersymmetric model with two Higgs doublets (MSSM), the coefficients are given by [2, 3, 4]

$$b_i = \left(\frac{33}{5}, 1, -3 \right), \quad b_{ij} = \begin{pmatrix} 199/25 & 27/5 & 88/5 \\ 9/5 & 25 & 24 \\ 11/5 & 9 & 14 \end{pmatrix}, \quad a_{ij} = \begin{pmatrix} 26/5 & 14/5 & 18/5 \\ 6 & 6 & 2 \\ 4 & 4 & 0 \end{pmatrix} \quad (2)$$

The two-loop gauge coupling terms involving the b_{ij} affect the prediction for $\alpha_3(M_Z)$ by $\approx 10\%$. The two-loop Yukawa coupling terms involving the a_{ij} affect the prediction for $\alpha_3(M_Z)$ by $\approx 1\%$. GUT scale threshold corrections can also affect the prediction

*Presented by MSB at the XXIII International Symposium on Multiparticle Dynamics, 12-17 September 1993, Aspen, Colorado.

for $\alpha_3(M_Z)$ [5].

2. Yukawa Coupling Evolution

At one-loop the particle content of the MSSM gives [6]

$$\frac{d\mathbf{U}}{dt} = \frac{1}{16\pi^2} \left[-\sum c_i g_i^2 + 3\mathbf{U}\mathbf{U}^\dagger + \mathbf{D}\mathbf{D}^\dagger + \mathbf{Tr}[3\mathbf{U}\mathbf{U}^\dagger] \right] \mathbf{U}, \quad (3)$$

$$\frac{d\mathbf{D}}{dt} = \frac{1}{16\pi^2} \left[-\sum c'_i g_i^2 + 3\mathbf{D}\mathbf{D}^\dagger + \mathbf{U}\mathbf{U}^\dagger + \mathbf{Tr}[3\mathbf{D}\mathbf{D}^\dagger + \mathbf{E}\mathbf{E}^\dagger] \right] \mathbf{D}, \quad (4)$$

$$\frac{d\mathbf{E}}{dt} = \frac{1}{16\pi^2} \left[-\sum c''_i g_i^2 + 3\mathbf{E}\mathbf{E}^\dagger + \mathbf{Tr}[3\mathbf{D}\mathbf{D}^\dagger + \mathbf{E}\mathbf{E}^\dagger] \right] \mathbf{E}, \quad (5)$$

where $c_i = (13/15, 3, 16/3)$, $c'_i = (7/15, 3, 16/3)$, $c''_i = (9/5, 3, 0)$. The two-loop equations in their full matrix form can be found in the appendix of Ref. [4]. The individual terms in these equations can be understood independently. The terms involving the gauge couplings arise from the contribution $c_i(f)$ to the anomalous dimension of each field in the Yukawa coupling. For example, $c_i = c(q_L) + c(u_R) + c(H_2)$ where $c_i(f)$ is $\frac{N^2-1}{N}$ (0) for the fundamental representation (singlet) of $SU(N)$ and $\frac{3}{10}Y^2$ (suitably normalized so that $Y_\tau = 2$) for $U(1)_Y$. Furthermore the trace contribution must arise from fermion loops as in Figure 1.

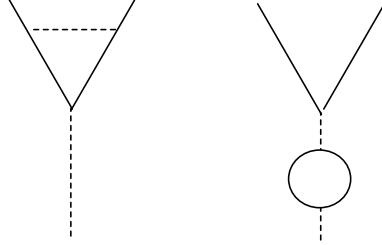


Fig. 1. Two of the diagrams which contribute to the one-loop Yukawa coupling renormalization group equations.

We say that a variable X scales when it obeys a differential equation of the form

$$\frac{dX}{dt} = \frac{X}{16\pi^2} [\dots]. \quad (6)$$

The gauge and Yukawa couplings are of this form to leading order in the fermion hierarchy. The scaling factors S_i for the fermion evolution may be defined as

$$\lambda_i(M_G) = S_i \lambda_i(m_t), \quad (7)$$

The S_i are plotted versus $\tan\beta$ for $m_t = 150$ GeV in Figure 2.

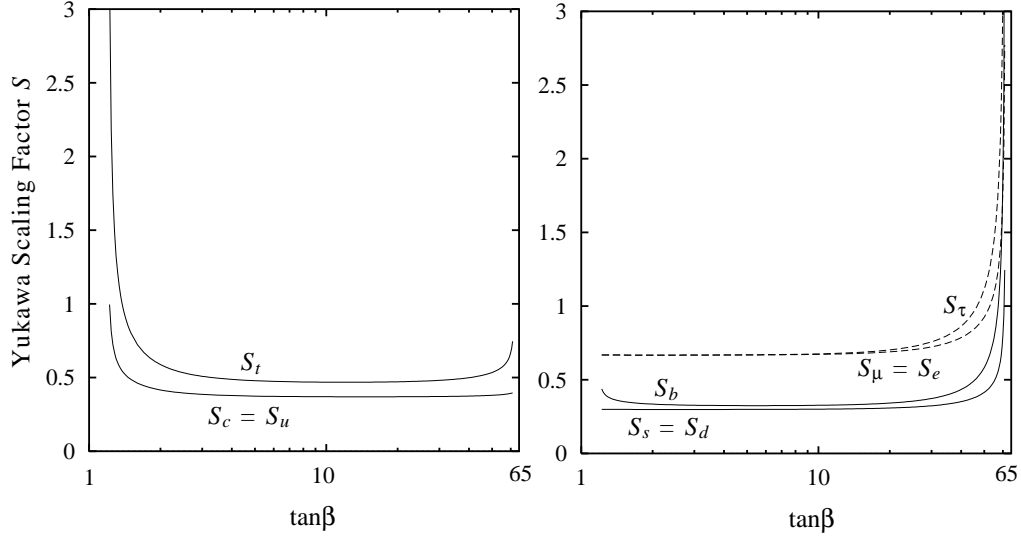


Fig. 2. Typical Yukawa scaling factors $S_i = \lambda_i(M_G)/\lambda_i(m_t)$ with $m_t = 150$ GeV.

The evolution of the gauge and Yukawa couplings (which are dimensionless parameters) does not depend on the soft-supersymmetry breaking parameters (which are dimensionful). The analysis decouples to first order from the details of SUSY breaking though the SUSY spectrum can still affect results through threshold effects. The evolution of the soft-supersymmetry breaking parameters do depend on dimensionless gauge and Yukawa couplings, however. For instance the right-handed soft top-squark mass $M_{\tilde{t}_R}$ has the RGE

$$\frac{dM_{\tilde{t}_R}^2}{dt} = \frac{2}{16\pi^2} \left(-\frac{16}{15}g_1^2M_1^2 - \frac{16}{3}g_3^2M_3^2 + 2\lambda_t^2X_t \right), \quad (8)$$

where $X_t = M_{Q_L}^2 + M_{\tilde{t}_R}^2 + M_{H_2}^2 + A_t^2$ is a combination of SUSY mass parameters. These soft-supersymmetry breaking parameters do not exhibit scaling (the supersymmetric Higgs mass parameter μ appearing in the superpotential does scale, however).

Grand unified theories give the boundary conditions to the above differential equations. The grand unified group guarantees certain relations between Yukawa couplings when the Higgs sector is required to be simple. The first such example was $\lambda_b = \lambda_\tau$ given by Chanowitz, Ellis, and Gaillard[7] in 1977. Georgi and Jarlskog[8] subsequently proposed viable relations for the lightest two generations: $3\lambda_s = \lambda_\mu$, $\frac{1}{3}\lambda_d = \lambda_e$.

3. Fixed Points

Yukawa couplings if large are driven to a fixed point at the electroweak scale. The Yukawa couplings are related to the fermions masses (in our convention) by

$$\lambda_b(m_t) = \frac{\sqrt{2}m_b(m_b)}{\eta_b v \cos \beta}, \quad \lambda_\tau(m_t) = \frac{\sqrt{2}m_\tau(m_\tau)}{\eta_\tau v \cos \beta}, \quad \lambda_t(m_t) = \frac{\sqrt{2}m_t(m_t)}{v \sin \beta}. \quad (9)$$

The scaling factors η_b and η_τ relate the Yukawa couplings to their values at the scale

m_t . The evolution of these Yukawa couplings can be deduced from Eqs. (3-5),

$$\frac{d\lambda_t}{dt} = \frac{\lambda_t}{16\pi^2} \left[-\frac{13}{15}g_1^2 - 3g_2^2 - \frac{16}{3}g_3^2 + 6\lambda_t^2 + \lambda_b^2 \right], \quad (10)$$

$$\frac{dR_{b/\tau}}{dt} = \frac{R_{b/\tau}}{16\pi^2} \left[-\frac{4}{3}g_1^2 - \frac{16}{3}g_3^2 + \lambda_t^2 + 3\lambda_b^2 - 3\lambda_\tau^2 \right]. \quad (11)$$

where we have defined $R_{b/\tau} \equiv \frac{\lambda_b}{\lambda_\tau}$. The behavior of the top quark Yukawa coupling is exhibited in Figure 3 assuming that the bottom quark and the tau lepton Yukawa couplings are approximately unified at a SUSY-GUT scale of approximately 2×10^{16} GeV. The coupling approaches a quasi-infrared fixed point[9] of approximately 1.1. We have taken different GUT scale threshold corrections for each curve.

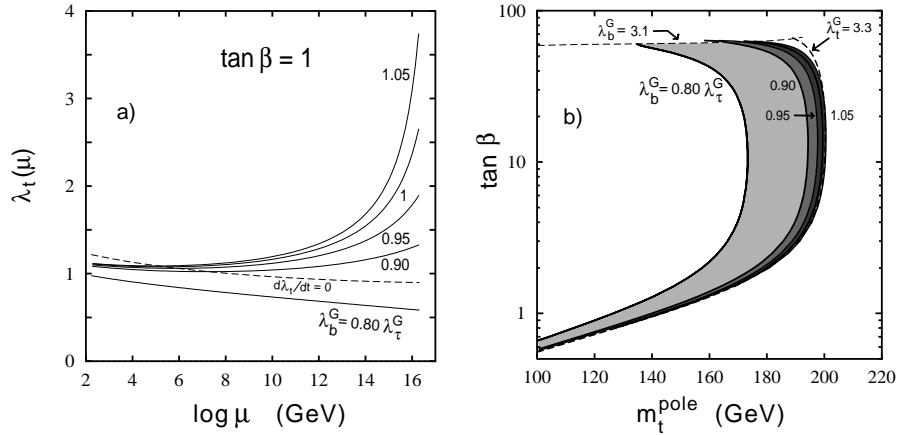


Fig. 3. a) The Yukawa coupling λ_t approaches a fixed point at the electroweak scale. All curves are attracted to the dashed line as the scale μ is decreased. b) Effects of GUT threshold corrections to Yukawa coupling unification. Here $\alpha_3(M_Z) = 0.118$ is assumed.

For $\lambda_b^G = 0.80\lambda_\tau^G$, the fixed point does not adequately describe the electroweak scale value of λ_t . This can be seen in Figure 3b by looking at the $\tan\beta = 1$ solutions in the $m_t, \tan\beta$ plane. For $\lambda_b^G = 0.80\lambda_\tau^G$, the predicted value of m_t is 20 GeV lower than the fixed point value. For larger values of $\alpha_3(M_Z)$ still larger threshold corrections would be necessary to avoid the fixed point.

The fixed point solution[4, 10, 11, 12, 13, 14] leads to the following relation between the \overline{DR} (dimensional reduction with minimal subtraction) top quark mass and $\tan\beta$

$$\lambda_t(m_t) = \frac{\sqrt{2}m_t(m_t)}{v \sin\beta} \Rightarrow m_t(m_t) \approx \frac{v}{\sqrt{2}} \sin\beta = (192\text{GeV}) \sin\beta, \quad (12)$$

Converting this relation for the top quark pole mass yields

$$m_t^{\text{pole}} = (200\text{GeV}) \sin\beta. \quad (13)$$

If one takes the λ_t fixed point solution seriously and also assumes that the top quark mass m_t^{pole} is less than about 160 GeV, important consequences result for the Higgs sector of the MSSM. From Figure 3b it is clear that given these assumptions $\tan\beta$ is very near one. Since $\tan\beta = 1$ is a flat direction in the Higgs potential, the

tree level mass is very small and the true mass of the lightest Higgs is given almost entirely by the one-loop radiative corrections. This case was discussed in detail by Diaz and Haber[15]. The upper bound that results is shown by the boundary of the theoretically disallowed region in Figure 4. We have made the assumption that colored SUSY particles are about 1 TeV to calculate the radiative corrections.

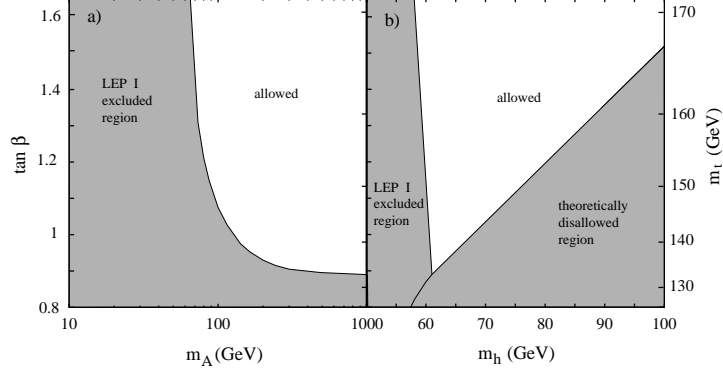


Fig. 4. The λ_t -fixed-point solution regions allowed by the LEP I data: (a) in the $(m_A, \tan \beta)$ plane, (b) in the $(m_h, \tan \beta)$ plane. The top quark masses are $m_t(\text{pole})$, correlated to $\tan \beta$ by the fixed point solution[13].

LEP II will be able to discover the lightest SUSY Higgs boson for m_t^{pole} up to 160 GeV provided the fixed point solution for the top Yukawa coupling is satisfied (see Figure 5). If $m_t \gtrsim 170$ GeV, $\tan \beta$ is not constrained.

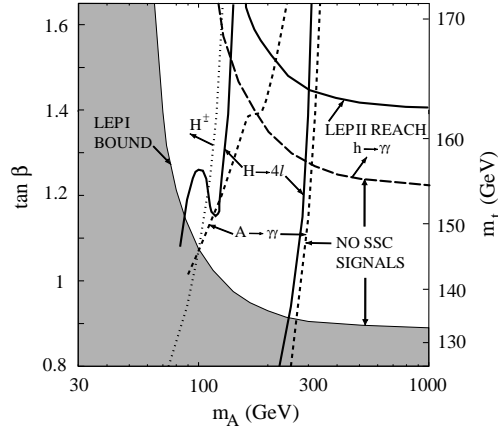


Fig. 5. Signal detectability regions, compared with the LEP I allowed region of λ_t -fixed-point solutions and the probable reach of LEP II[13]. The top quark masses are $m_t(\text{pole})$.

4. Universal Scaling of the CKM Matrix

There are three “kinds” of CKM matrix elements in regards to scaling behavior: (1) diagonal, (2) mixing - between heavy and light generations, e.g. V_{cb} , (3) mixing - between two light generations, e.g. V_{us} . The evolution equations for the Yukawa couplings lead immediately to evolution equations for the mixing angles in the CKM matrix:

$$\frac{d\mathbf{U}}{dt}, \frac{d\mathbf{D}}{dt} \Rightarrow \frac{dV_{\text{CKM}}}{dt}, \quad (14)$$

Provided the mixings between heavy and light generations are small one can prove that there are only two types of scaling to leading order in the fermion hierarchy[16, 17, 18]

$$\frac{d|V_1|^2}{dt} = \frac{d|V_3|^2}{dt} = 0 , \quad (15)$$

$$\frac{d|V_2|^2}{dt} = -\frac{|V_2|^2}{8\pi^2} (a_d\lambda_t^2 + a_u\lambda_b^2) + 2 - \text{loop} . \quad (16)$$

We highlight some features of the evolution:

- Gauge couplings contributions do not appear in the RGEs of the CKM elements.
- The approximation of scaling is particularly good even though the Cabibbo angle $|V_{us}|$ is not small.
- The scaling behavior is a property of the hierarchy; it can be proven to all orders in perturbation theory.
- The universality of the scaling is model-independent. However the amount of the scaling varies between various models, For example in the MSSM one has $a_u = a_d = 1$ while in the Standard Model $a_u = a_d = -\frac{3}{2}$.

The evolution of the CKM matrix is important when one examines the relations between masses and mixings. One example that has been thoroughly examined recently[4, 19, 20, 21, 22] is

$$|V_{cb}| = X \sqrt{\frac{\lambda_c}{\lambda_t}} \quad \text{at } M_G . \quad (17)$$

where X can account for GUT scale threshold corrections, or for Clebsch factors[22]. Some other relations also occur under rather general assumptions[23].

5. Acknowledgements

This research was supported in part by the University of Wisconsin Research Committee with funds granted by the Wisconsin Alumni Research Foundation, in part by the U.S. Department of Energy under contract no. DE-AC02-76ER00881, and in part by the Texas National Laboratory Research Commission under grant nos. RGFY93-221 and FCFY9302. MSB was supported in part by an SSC Fellowship. PO was supported in part by an NSF Graduate Fellowship.

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